

Research Article

New Existence Results for Nonlinear Fractional Differential Equations with Three-Point Integral Boundary Conditions

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This paper studies a boundary value problem of nonlinear fractional differential equations of order $q \in (1, 2]$ with three-point integral boundary conditions. Some new existence and uniqueness results are obtained by using standard fixed point theorems and Leray-Schauder degree theory. Our results are new in the sense that the nonlocal parameter in three-point integral boundary conditions appears in the integral part of the conditions in contrast to the available literature on three-point boundary value problems which deals with the three-point boundary conditions restrictions on the solution or gradient of the solution of the problem. Some illustrative examples are also discussed.

1. Introduction

In recent years, boundary value problems for nonlinear fractional differential equations have been addressed by several researchers. Fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes; see [1]. These characteristics of the fractional derivatives make the fractional-order models more realistic and practical than the classical integer-order models. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, and fitting of experimental data, [1–4]. For some recent development on the topic, see [5–21] and the references therein.